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THEORETICAL EVALUATION OF SPECIFIC HEAT CAPACITY AND ITS APPLICATION OF SOME SOLAR CELL MATERIALS

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ABSTRACT

A simple and efficient analytical formula for the Debye function has been derived based on the binomial expression and gamma function in this study. Different analytic formulae for Debye function are given, and the convergence of the corresponding series is discussed for a wide range of the parameters. In this study, the analytic formula presented for the Debye function can be applied to evaluate some of the thermodynamic properties of solid. It has been demonstrated that this formula gives very accurate results for the specific heat capacity of solar cells over a wide range of temperatures. The results for specific heat capacity of solar cell Si, $CuInSe_2$ and CdTe materials have been compared with theoretical calculations and experimental data and indicated that the analytical formula can be satisfactorily used for other solids.

1. INTRODUCTION

The accurate determination of thermodynamic properties of solids, such as entropy, internal energy, and specific heat capacity, is of fundamental importance in condensed matter physics and material science. Many authors have calculated theoretically basic physical properties of matter such as electronic structure under different pressure, elastic and thermodynamic properties and specific heat capacity [1-3]. Among various theoretical models, the Debye model has been widely employed due to its ability to describe vibrational spectra and related thermodynamic quantities of solids more successfully than the Einstein model, particularly at low temperatures. To determine the thermodynamic properties of solid, the Debye function has been considered in many studies including analytical, numerical, and experimental methods [4-7]. The Debye function, based on the Debye temperature, provides a powerful framework for evaluating several thermal properties of solids, including specific heat capacity [8-9]. Numerous studies have applied analytical, numerical, and experimental methods to calculate Debye functions and related thermophysical properties [10-14]. Also, the definition of the Debye function is a significant and influential numerical method for evaluating the thermophysical properties of solid-state solar cells, which are typically made of semiconductor materials. Therefore, semiconductors have a special importance in the construction of solar cells. It is very important to accurately and precisely analyze the thermodynamic properties of semiconductors used in solar cell production. Variations in temperature strongly influence device performance, as an increase in temperature usually enhances current but decreases voltage, thereby reducing efficiency. Consequently, a reliable description of the specific heat capacity of semiconductor-based solar cells is essential for predicting their performance under different operating conditions.

It is well known, solar cells began to be developed for space programs, but in the following years, solar cells started to be used in places such as lighthouses, forest watchtowers, and farmhouses, chalets where electricity generation is difficult or remote in known ways [15-19]. Also, many materials have suggested to product solar cells such as Si, Ge, GaAs, CdTe and $CuInSe_2$, $Cu(InGa)Se_2$in this field [20-21]. Si solar cells play an important role the photovoltaic industry as state-of-the-art devices with strong future potential [22-23]. $CuInSe_2$ solar cells have major potential photovoltaic applications field [24-25]. CdTe

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solar cell technology currently holds a major market share and has attached significant attention over nowadays and has admitted a high level of attention from numerous technology companies [26-27]. Also, CdTe solar cell has been identified as a strong candidate for thin film solar cell implementation [28].

Many authors have proposed experimental and theoretical methods to develop solar cells for various applications. Iqbal et al. have examined thermoelectric characteristics of solar cell energy sources by ab-initio computations [29]. Chaudhry et al. have investigated thin film GaAs solar cells with dielectric nanoparticles [30]. Hu has improved efficiently perovskite solar cells and achieved a maximum power conversion efficiency of %26.11 [31]. Bati et al. have studied photovoltaic-integrated technologies [32]. Ferreira and Moreira have offered theoretical model for molar-specific heat capacity of solids to shown heat capacity dependence on nucleation kinetics [33]. Eser et al. have suggested an analytical method for calculate thermophysical properties of solids such as solar cell using Debye-functions [34-35]. Therefore, efficient thermophysical studies of solar cells depend on the accuracy of the n-Debye function.

The goal of the paper is to offer an efficient and simple analytical formula for the calculation of specific heat capacity with an n-dimensional Debye function in a wide range of temperature ranges. The proposed method, which notably developed its ability during analytical evaluations significant state, is proven as a correct and available method of computation of Debye functions of solids by applying it to solar cell Si, $CuInSe_2$, and CdTe materials.

2. MATERIAL and METHOD

2.1. Theory and Basic Formula

The thermal energy of matter is distributed among several modes such as translational, vibrational, rotational, electronic, and magnetic contributions [33]:

$$C_v = C_v^{Trans} + C_v^{Vib} + C_v^{Rot} + C_v^{e} + C_v^{Mag} \tag{1}$$

Here, Einstein's model does not accurately predict to experimental results at low temperatures. Debye model defines properties such as vibrational spectra and harmonic frequencies, which are expressed in term of Debye's frequency [33]. The integer and non-integer Debye functions are expressed as follows [36-37]:

$$D_n\left(\beta, x_D\right) = \frac{n}{x_D^n} \int_0^{x_D} \frac{t^n}{\left(e^t - 1\right)^{\beta}} dt \tag{2}$$

where n, β is integer and non-integer, $x_D = \theta/T$ is the ratio of Debye temperature to a temperature and θ is Debye temperature and T is temperature.

For the analytical assessment of the n- dimensional Debye functions, we used binomial expression theorem, expressed as follows:

$$\left(x \pm y\right)^n = \sum_{j=0}^{\infty} \left(\pm 1\right)^j F_j\left(n\right) x^{n-j} y^j \tag{3}$$

where $F_j\left(n\right)$ is binomial coefficient expressed by [38-39]:

$$F_j\left(n\right) = \begin{cases} n! \Big/ \left[j!\left(n-j\right)! \right] & for \ \text{integer } n \\ \left(-1\right)^j \Gamma\left(j-n\right) \Big/ j! \Gamma\left(-n\right) & for \ non\text{-integer } n \end{cases} \tag{4}$$

Here $\Gamma(n)$ represents gamma function. By substituting Eq. (3) into Eq. (2), we obtained the analytical formula for n-Debye functions in the following form:

$$D_n(\beta, x_D) = \frac{n}{x_D^n} \lim_{N\to\infty} \sum_{m=0}^{N} (-1)^{m-2n-1} F_m(-\beta)(\beta+m)^{-n-1} \left(-n\,\Gamma(n) + \Gamma(n+1, x_D(\beta+m))\right) \quad (5)$$

Here, n and β are positive integer , $F_j(n)$ is the binomial constant, is the ratio of Debye temperature to a temperature, $\Gamma(n)$ and $\Gamma(\alpha, x)$ are gamma function and incomplete gamma function, respectively [38-39]. In Eq. (5) the indices N is the summation upper limits.

2.2. Definition of specific heat capacity according to Debye functions

The specific heat capacity of a solid is written using Debye approximation as follows [40-41]:

$$C_V = 3N_A k_B L_V(x_D) \quad (6)$$

where N_A is the Avogadro constant, k_B is the Boltzmann constant, and $L_V(x_D)$ is an auxiliary function expressed in term of Debye functions as follows [40-41]:

$$L_V(x_D) = (n+1) D_n(1, x_D) - \left(\frac{n\,x_D}{e^{x_D} - 1}\right) \quad (7)$$

By substituting Eq. (5) into Eq. (7) and Eq. (6) we obtain the analytical formula for the specific heat capacity. As seen from Eq. (6), for the evaluation of the specific heat capacity of solids it is required to the precise determination of Debye functions. The specific heat capacity of a solid is defined in this work as follows:

$$C_V = 3\,N_A k_B\,(n+1) D_n(1, x_D) - \left(\frac{n\,x_D}{e^{x_D} - 1}\right). \quad (8)$$

3. NUMERICAL RESULTS AND DISCUSSION

In this study, a simple and valid analytical formula is proposed for the direct assessment of n-dimensional Debye functions over a wide range of integer and non-integer values of β and x_D. The analytical expression was evaluated by solving Eq. (2) which can be computed easily and quickly using the binomial series expansion theorem. Mathematica 7.0 program was used to calculate the Debye functions with the proposed analytical formula. To show the correctness and influence of the method defined above, we offer several numerical results in Table 1. In Table 1, the accuracy of the obtained analytical expression is demonstrated by comparison with other studies. This confirms the efficiency of the binomial expansion approach, which requires fewer terms for accurate results compared to other approximation techniques. For non-integer n values, the convergence is slightly slower, but the proposed method still provides results that are sufficiently accurate for practical purposes. As seen in Table 1, the analytical formula is sufficiently accurate for arbitrary parameter values. The results obtained in Table 1, demonstrate that the analytical formula is in good agreement with literature data [42]. Small deviations occur particularly at high-precision decimal places and are due to differences in the numerical method used Mathematica's integral-based solution vs. the binomial expansion + gamma function approach used in the study. The analytical formula proposed in the study shows almost one-to-one agreement with Mathematica's high-precision numerical calculations. This demonstrates the formula's reliability and rapid computational advantage. Therefore, the analytical method offers superior accuracy and ease of calculation when compared to experimental and theoretical literature data.

Table 1. Comparative values of n-dimensional Debye functions

n	s	T	N	This work	Mathematica numeric results	Ref.[42]
2	1	5	800	0.172329034857624782145	0.1759741179993409	0.172329034857624782145
5	1	4.5	800	0.10164118339698890967	0.10164118339698923	0.10164118339698890967
7	1	0.8	800	0.69112406526865230673	0.6911240652686543	0.69112406526865230673
9	1	3.4	800	0.15413773867789254146	0.1541377386778928	0.15413773867789254146
12	1	5.4	800	0.036188492338281326047	0.036188492338281426	0.036188492338281326047
15	1	2.4	300	0.26614091566472949520	0.2661409156647299	0.2661409156647294951955
20	1	1.24	300	0.52336158509003284377	0.5233615850900337	0.523361585088859680745
25	1	4.2	100	0.072964967062185871090	0.07296496706218619	0.0729649670621858713684
30	1	3.42	100	0.12584261065903688395	0.12584261065903765	0.125842610659065566078 1
21	10	18	800	$0.23506677124764645229\times10^{-26}$	$2.350667712476428\times10^{-27}$	$0.2350667712476464522 9\times10^{-26}$
25	4.5	8.4	200	$3.171488338588378 1453\times10^{-14}$	$3.171488338588358\times10^{-14}$	$3.171488338588378 14525\times10^{-14}$

Table 2 indicates that the convergence properties of Eq. (5) vary significantly. As seen from Table 2, the advantage of this study is that Eq. (5) shows faster convergence to the numerical results for $n>8$ integer values compared to other studies.

Table 2. Convergence of derived expression for different parameter values n-dimensional Debye functions as a function of summation limit N

N	$D_{10}(1, 2.6)$	$D_{24}(6.5, 4.3)$	$D_{14}(1, 7)$
50	0.24784443192315386611	$4.1670806873024402544\times10^{-11}$	0.010312098488746688362
60	0.24784443192315548968	$4.1670806873024406529\times10^{-11}$	0.010312098488746688362
70	0.24784443192315574783	$4.1670806873024406734\times10^{-11}$	0.010312098488746688362
80	0.24784443192315580151	$4.1670806873024406750\times10^{-11}$	
85	0.24784443192315581043	$4.1670806873024406751\times10^{-11}$	
90	0.24784443192315581514	$4.1670806873024406751\times10^{-11}$	
95	0.24784443192315581771	$4.1670806873024406751\times10^{-11}$	
100	0.24784443192315581917	$4.1670806873024406751\times10^{-11}$	
120	0.24784443192315582102		
140	0.24784443192315582130		
160	0.24784443192315582136		
170	0.24784443192315582137		
175	0.24784443192315582137		

Therefore, this analytical formula provides sufficiently accurate results over a wide range of parameters for the n-Debye functions. As an example of specific heat capacity for some solar cell materials Si, $CuInSe_2$, and CdTe are given to show the validity of the suggested method in Tables 3-5.The results were compared with both theoretical models and available experimental data. In Tables 3–5, the accuracy and precision of the analytical expression for specific heat capacity are demonstrated through comparisons with theoretical and experimental data for $n=3$ and $n=2$ values [43-44]. As seen in Tables 3–5, the analytical formula for specific heat capacity is sufficiently accurate over a wide temperature range. The results presented in Tables 3–5 and Figures 1–3 show good agreement with both theoretical and experimental data [43-45]. In Table 3, the calculation results of the heat capacity of Si for the $\theta=645[K]$ Debye temperature is in good agreement with the literature for $n=2$ value. Also, as seen from Table 3 and Fig. 1, the calculation results for the heat capacity of Si in $n=3$ and $n=2$ values are in good compatible with the experimental data than Ref. [43].

Table 3. Specific heat capacity of Si for $\theta = 645\ [K]$ Debye temperature

T [K]	This work		Ref. [43]	Standard deviation	Standard deviation	Experiment Ref. [43]	Standard deviation	Standard deviation
	n=3	n=2		n=3	n=2		n=3	n=2
22.7	0.0847316	0.445652	0.796	0.502943	0.247733	-		
29.5	0.185965	0.752641	0.970	0.554396	0.153696	0.623	0.30903	0.09167
30.3	0.201508	0.794015	1.43	0.868675	0.449709	-		
41.6	0.52121	1.49651	2.55	1.43457	0.74493	-		
49.0	0.84961	2.07492	2.77	1.35792	0.491496	2.15	0.919515	0.0530896
51.0	0.956625	2.2469	3.11	1.52267	0.610304	-		
60.5	1.57602	3.1489	4.22	1.86958	0.757382	-		
71.8	2.54369	4.37887	5.18	1.86415	0.566484	4.44	1.34089	0.0432254
81.2	3.50936	5.49344	5.90	1.69044	0.287481	-		
90.6	4.57832	6.65135	6.85	1.60632	0.140467	-		
100	5.70736	7.81771	7.49	1.26052	0.231726	7.25	1.09081	0.401432
110	6.93077	9.03574	8.45	1.07426	0.414181	-		
121	8.25961	10.3192	8.93	0.474037	0.982313	-		
130	9.30955	11.3107	9.80	0.346801	1.06823	-		
140	10.4203	12.3421	10.3	0.0850649	1.44398	-		
151	11.5629	13.3874	11.2	0.256609	1.54673	-		
160	12.4321	14.1734	11.5	0.659094	1.89038	12.8	0.260145	0.97114
200	15.6015	16.9848	14.1	1.06172	2.03986	15.5	0.0717713	1.04991
251	18.3176	19.3407	16.6	1.21453	1.93797	-		
261	18.7218	19.6879	17.0	1.2175	1.90063	18.6	0.0861256	0.769261
301	20.0383	20.8131	18.4	1.15845	1.70632	20.2	0.114339	0.433527
350	21.1726	21.7765	20.2	0.687732	1.11475			
401	21.9928	22.4697	21.4	0.419173	0.756392			
450	22.558	22.9459	22.6	0.0296985	0.244588			
501	22.9927	23.3114	23.3	0.217294	0.00806102			
551	23.3147	23.5817	23.9	0.41387	0.225072			
600	23.5597	23.787	24.1	0.38205	0.221324			
651	23.7609	23.9555	24.2	0.310491	0.172888			
700	23.9159	24.0852	24.3	0.2716	0.151887			
750	24.0449	24.1931	24.4	0.251094	0.1463			
801	24.1532	24.2835	24.4	0.174514	0.0823779			
852	24.2431	24.3586	24.4	0.110945	0.0292742			
900	24.3145	24.4182	24.4	0.0604576	0.0128693			
951	24.3791	24.4721	24.4	0.0147785	0.0509824			
1000	24.4323	24.5165	24.5	0.0478711	0.0116673			

The obtained numerical results for n=3 and n=2 values from Eq. (4), theoretical and experimental data are plotted in Figure 1. It can be seen from graphics that obtained results Eq. (5) are well in agreement with experimental data. Figure 1 indicates that the suggested method may lead to a very more efficient algorithm for the correct and sensitive estimation of the specific heat capacity.

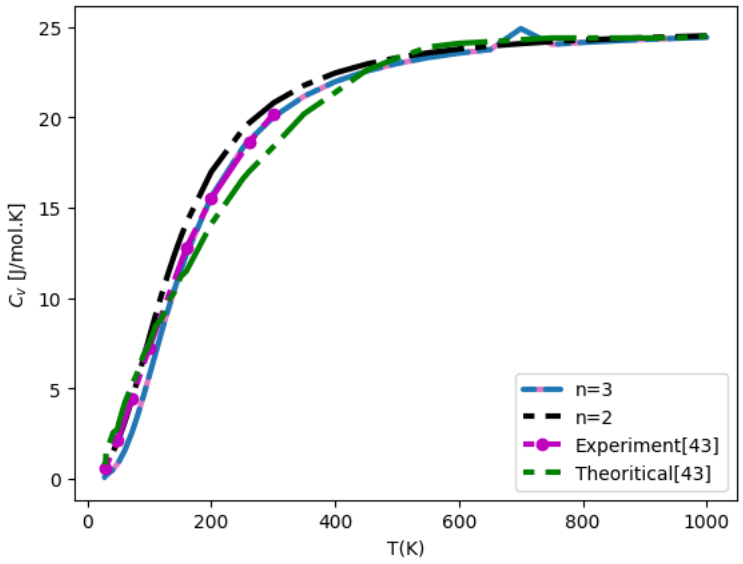

Figure 1. The temperature dependence of heat capacity of Si ($\theta = 645$ [K])

In Table 4, the calculation results of the heat capacity of $CuInSe_2$ for the $\theta = 225$ [K] Debye temperature are in good compatible with the literature for $n=3$ value. Also, the heat capacities of $CuInSe_2$ are compared with experimental data [44] at different range temperature. As can be seen Table 4, the obtained results for specific heat capacity of $CuInSe_2$ are in good agreement with experimental data [44].

Table 4. Specific heat capacity of $CuInSe_2$ for $\theta = 225$ [K] Debye temperature

T [K]	This work		Ref. [44]	Standard deviation	Standard deviation	T [K]	This work		Experiment [44]	Standard deviation	Standard deviation
	n=3	n=2		n=3	n=2		n=3	n=2		n=3	n=2
11.925	0.289373	1.01068	0.34422	0.0387827	0.471258	9.2475	0.13495	0.607781	0.493881	0.253803	0.0805395
19.35	1.22511	2.65434	1.74854	0.370121	0.640497	16.1775	0.721203	1.85925	1.79344	0.758186	0.0465347
25.425	2.64851	4.5041	2.94333	0.208469	1.10363	18.495	1.07285	2.42705	2.66895	1.12861	0.171049
30.375	4.16813	6.21438	4.34017	0.121651	1.32527	19.845	1.31889	2.79012	2.96828	1.16629	0.125978
34.65	5.62576	7.73495	5.61229	0.00952473	1.50095	23.625	2.1677	3.91861	4.04085	1.32452	0.0864367
38.475	6.96681	9.07105	7.23361	0.188656	1.29927	24.75	2.46292	4.28151	4.43994	1.39796	0.112027
42.075	8.21381	10.2755	8.55562	0.241696	1.21614	29.475	3.87454	5.89628	5.61229	1.22877	0.200811
45.675	9.41607	11.4103	9.75291	0.238182	1.17195	30.6	4.24246	6.29417	6.21093	1.39192	0.0588596
49.95	10.7605	12.6549	11.0999	0.239992	1.09955	35.55	5.94048	8.05303	6.83452	0.632182	0.861617
54.9	12.1852	13.9509	12.5715	0.273155	0.975383	46.35	9.63486	11.6145	9.0046	0.445661	1.84548
60.975	13.7321	15.336	13.9684	0.167089	0.967039						
67.95	15.2482	16.6752	15.3652	0.0827315	0.92631						
74.025	16.3675	17.6536	16.6373	0.190777	0.718633						
82.125	17.7598	17.6138	18.7343	0.689076	0.792313						
90.9	18.7056	19.674	18.9571	0.177837	0.506925						
99.675	19.5863	20.4278	19.7303	0.101823	0.493207						
108.9	20.336	21.0665	20.4786	0.100833	0.415708						
115.2	20.7653	21.4312	20.7653	0.0000	0.470862						
123.3	21.238	21.8319	21.4015	0.115612	0.304339						
141.525	22.0551	22.5223	22.2496	0.137532	0.192828						
149.625	22.3361	22.7591	22.524	0.132865	0.166241						
159.75	22.6346	23.0104	22.7984	0.115824	0.149907						
171.0	22.911	23.2427	22.9979	0.0614476	0.1731						
182.925	23.154	23.4468	23.3471	0.136542	0.0704985						
194.175	23.346	23.6079	23.4968	0.106632	0.0785596						
205.425	23.5091	23.7446	23.6963	0.13237	0.0341533						

continued Table 4.

T [K]	This work		Ref. [44]	Standard deviation	Standard deviation
	n=3	n=2		n=3	n=2
219.375	23.6793	23.8872	23.7462	0.0473054	0.0997021
231.75	23.8064	23.9935	23.8959	0.0632861	0.0690136
245.25	23.9245	24.0924	24.0206	0.067953	0.0507703
258.75	24.0253	24.1767	24.0954	0.0495682	0.0574878
272.25	24.112	24.2491	24.2201	0.0764382	0.0205061
283.5	24.1753	24.302	24.2451	0.0493561	0.0402344
294.75	24.2316	24.349	24.295	0.0448306	0.0381838
308.25	24.2914	24.399	24.3199	0.0201525	0.0559321
319.5	24.3358	24.436	24.3449	0.00643467	0.0644174
335.25	24.3906	24.4818	24.3698	0.0147078	0.079196

The obtained numerical results for n=3 and n=2 values from Eq. (4), theoretical and experimental data are plotted in Figures 2-3. It can be seen from graphics that obtained results Eq. (5) are well in agreement with experimental data. Figures 2–3 indicate that the suggested method may lead to a more efficient algorithm for the accurate and sensitive estimation of the specific heat capacity.

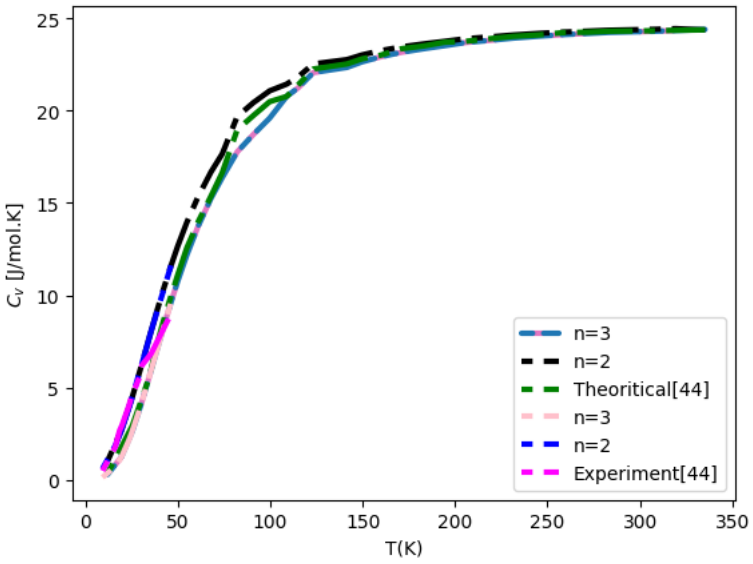

Figure 2.The temperature dependence of heat capacity of $CuInSe_2$ ($\theta = 225$ [K])

As seen from Table 5, the obtained results for specific heat capacity of CdTe in $n=3$ and $n=2$ values are in good agreement with theoretical and experimental data [45]. The standard deviation is given for the specific heat capacities of Si, $CuInSe_2$ and CdTe materials in Tables 3-5. To knowledge, standard deviation is the most widely used measure of variability. Standard deviation represents the typical distance between each data point and the mean. Smaller values indicate that the data points cluster closer to the mean—the values in the dataset are relatively consistent. As seen from Table 3, standard deviation values for n=2 is smaller than n=3. As seen from Table 4, standard deviation values for n=3 is smaller than n=2. But, considering experimental data, standard deviation values for n=2 is smaller than n=3 values. As seen from Table 5, standard deviation values for n=3 is smaller than n=2 values at low temperature. But, considering experimental data, standard deviation values for n=2 is smaller than n=3 values. The analytical expression suggested is a confident method to calculate the thermodynamic properties of solid and it provides great advantages according to its computational impact and its suitability. For this reason, the results of our calculations will be beneficial for different aspects of technology and industry. As an example, molecules are widely used to create solar panels. The use of these molecules increases the efficiency of the solar panel and helps to drive down costs and allow

utilities to trust on solar for baseload power. Therefore, the recommend analytical formula can be useful to determine the thermodynamic properties of solar cells used in this aim. Finally, it should be noted that while the present study has been limited to three representative solar cell materials, the approach can be generalized to other semiconductors or solid-state systems. The methodology is not restricted by crystal structure or composition, which means it can be extended to emerging materials such as perovskites, GaN, or organic-inorganic hybrid semiconductors. Future work may focus on incorporating anharmonic effects and temperature-dependent Debye temperatures to further improve predictive capability.

In conclusion, the numerical results and comparisons clearly demonstrate that the proposed analytical expression is accurate, efficient, and broadly applicable. It successfully bridges the gap between purely numerical methods and overly simplified models, offering a reliable tool for both theoretical studies and practical engineering applications.

Table 5. Specific heat capacity of CdTe for $\theta = 140$ $[K]$ Debye temperature

T [K]	This work		Ref. [45]	Standard deviation for	Standard deviation for	Experimental [45]	Standard deviation for	Standard deviation for
	n=3	n=2		n=3	n=2		n=3	n=2
16.7	3.05626	4.98063	-	-	-	8.26	3.6796	2.31886
28.2	9.30057	11.3023	-	-	-	10.8	1.06026	0.35518
37.8	13.6787	15.2885	-	-	-	13.2	0.338492	1.47679
48.6	17.0277	18.2272	-	-	-	15.4	1.15096	1.99913
58.7	19.0731	19.9889	-	-	-	17.1	1.39519	2.04276
69.5	20.5339	21.2347	-	-	-	18.6	1.36747	1.86301
79.7	21.4822	22.0385	-	-	-	20.1	0.977363	1.37073
89.8	22.1569	22.6081	-	-	-	20.9	0.888763	1.20781
99.9	22.6561	23.0285	20.2	1.73672	2.00005	21.9	1.24175	1.50508
100	22.6603	23.0321	20.3	1.66898	1.93189	-	-	-
120	23.3251	23.5904	21.4	1.36125	1.54885	23.4	0.0529623	0.134633
140	23.7395	23.9376	22.4	0.94717	1.08725	23.4	0.240063	0.380141
160	24.0141	24.1673	23.2	0.575656	0.683984	24.0	0.00997021	0.118299
180	24.205	24.3268	23.7	0.357089	0.443215	24.3	0.0671751	0.0189505
200	24.3429	24.442	24.1	0.171756	0.241831	24.7	0.252508	0.182434
220	24.4457	24.5277	24.4	0.0323148	0.0902975	24.8	0.250528	0.192545
240	24.5242	24.5931	24.5	0.017112	0.0658316	24.9	0.265731	0.217011
260	24.5856	24.6442	24.6	0.0101823	0.0312541	25.2	0.434446	0.39301
280	24.6345	24.6849	24.8	0.117026	0.081388	25.2	0.399869	0.364231
300	24.674	24.7177	25.0	0.230517	0.199616	25.3	0.47058	0.434941
330	24.7205	24.7563	25.1	0.268347	0.243033			
350	24.7451	24.7766	25.2	0.321663	0.299389			
380	24.775	24.8013	25.3	0.371231	0.352634			
400	24.7914	24.8148	25.4	0.430345	0.413799			
430	24.8118	24.8315	25.5	0.486631	0.472701			
450	24.8232	24.8408	25.6	0.549281	0.536835			
480	24.8377	24.8526	25.6	0.539027	0.528492			
500	24.8459	24.8593	25.6	0.533229	0.523754			

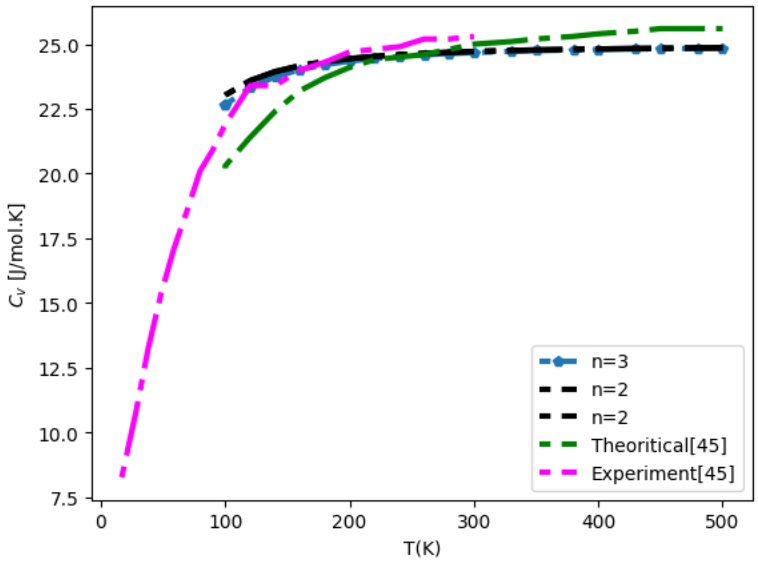

Figure 3. The temperature dependence of heat capacity of CdTe ($\theta = 140$ $[K]$)

Conflict of Interest Statement

There is no conflict of interest between the authors.

Statement of Research and Publication Ethics

The study is complied with research and publication ethics.

Artificial Intelligence (AI) Contribution Statement

This manuscript was entirely written, edited, analyzed, and prepared without the assistance of any artificial intelligence (AI) tools. All content, including text, data analysis, and figures, was solely generated by the authors.

Contributions of the Authors

The methodology, mathematical calculations, and analyses were also performed by Elif Somuncu and Ahmet Bulut.

Elif Somuncu and Ahmet Bulut were responsible for writing and editing the article.

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